EXPERIMENTAL INVESTIGATION INTO COEFFICIENTS OF MOMENTUM FLOW AND FRICTION ACCOMPANYING A UNIFORM OUTFLOW IN A POROUS PIPE

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The results of an experimental investigation into a turbulent flow in a circular pipe with a uniform spread through a permeable wall are presented. A calculation formula for determining the coefficient of friction is proposed. The nature of the variation in the coefficient of momentum flow is established.

It has become generally accepted that the hydrodynamics of a vapor flow in thermal pipes are identical to the course of a flow in porous channels with a uniform outflow (inflow) along the length [1, 2].

The variation in pressure in a porous channel accompanying a turbulent flow is described by an equation of motion with a variable flow rate along the length which takes the following form for a one-dimensional model [3]:

$$\frac{dp}{\rho} + \beta \bar{u} d\bar{u} + \bar{u}^2 d\beta + \beta \frac{\bar{u}(\bar{u} - \theta)\, dG}{G} + \xi \frac{u^2}{2} \cdot \frac{dx}{D} = 0. \quad (1)$$

Data on the variation in β, θ, and ξ along the porous channel are essential to the integration of Eq. (1). To this end an investigation is carried out in a porous pipe made from a perforated brass strip 0.25 mm thick with 50% porosity. The internal diameter of the pipe is 26.5 mm and the length of the porous section is 435 mm (16.4 in diameter) and that of the stabilization section is 630 mm (24 in diameter). The porosity is generated by 0.8-mm-diameter apertures arranged in staggered rows.

The experiments are carried out on the pneumatic test bench represented schematically in Fig. 1. The porous pipe is subdivided by brass disks (crosspieces) into separate sections 39.5 mm long (~1.5D) in order to produce a uniform outflow, and the resistance of the separate branches is increased by the use of throttle washers fitted to the outlet branches. The washer dimensions are selected on the basis of the condition that the pressure gradient at the washer be 25-30 velocity heads at the inlet to the porous pipe. In practice, this ensures that the flow spread along the working section is uniform. The longitudinal components of the velocity and static pressure in the various different sections of the porous pipe are measured by a special probe with 10 small total-pressure pipes located at equal intervals $(r/R)^2$ along both the radii and one small static-pressure pipe located at the center of the pipe. The small total- and static-pressure pipes are led off into a common guide pipe 5 mm in diameter which is located along the axis of the experimental section. The shading of the flow by the measuring probe is about 1% at the pressure sampling points and 8% at the guide bar installation point, which has no significant influence on the measurement accuracy.

The longitudinal components of the velocity are measured at five points in a given cross section at equal intervals $(r/R)^2$ in two mutually perpendicular planes.

The tests are made under conditions of a turbulent isothermal flow of air over a range of variations in the Reynolds number in front of the inlet to the porous section $Re_0 = (5.29\text{-}6.72) \cdot 10^4$.

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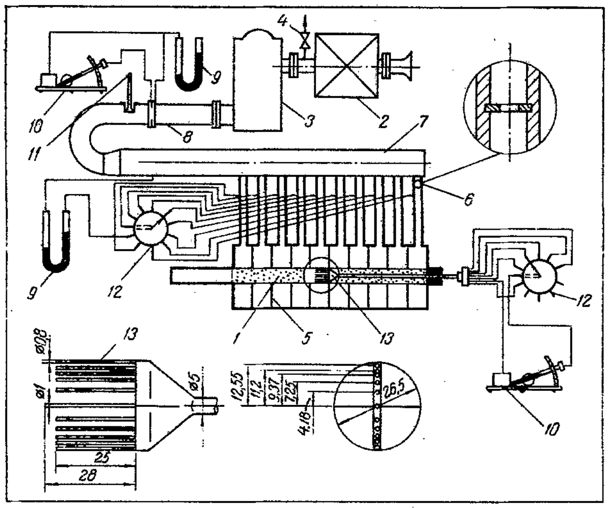

Fig. 1. Experimental test bench and equipment: 1) porous pipe; 2) blower; 3) receiver; 4) throttle valve; 5) crosspiece; 6) throttle washer; 7) collector; 8) measurement diaphragm; 9) manometer; (10) micromanometer; 11) thermometer; 12) tapswitch; 13) probe.

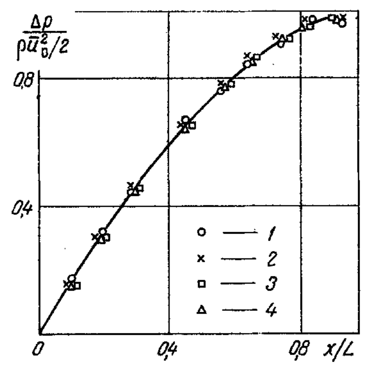

Fig. 2

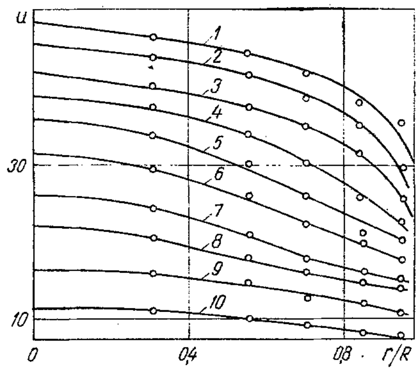

Fig. 3

Fig. 2. Dependence of $\frac{\Delta P}{\rho \frac{\bar{u}_0^2}{2}}$ on $\frac{x}{L}$: 1) $Re_0 = 6.72 \cdot 10^4$; 2) $5.29 \cdot 10^4$; 3) $3.11 \cdot 10^4$; 4) $4.41 \cdot 10^4$.

Fig. 3. Variation in axial velocity u(m/sec) from r/R and x/D: 1) $\frac{x}{D} = 0$; 2) 1.5; 3) 3.0; 4) 4.5; 5) 6.0; 6) 7.5; 7) 9.0; 8) 10.5; 9) 12.0; 10) 13.5. $Re_0 = 67{,}200$.

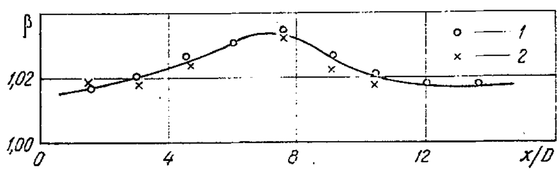

Fig. 4. Dependence of coefficient of momentum flow β on x/D: 1) $Re_0 = 67{,}200$; 2) 52,900.

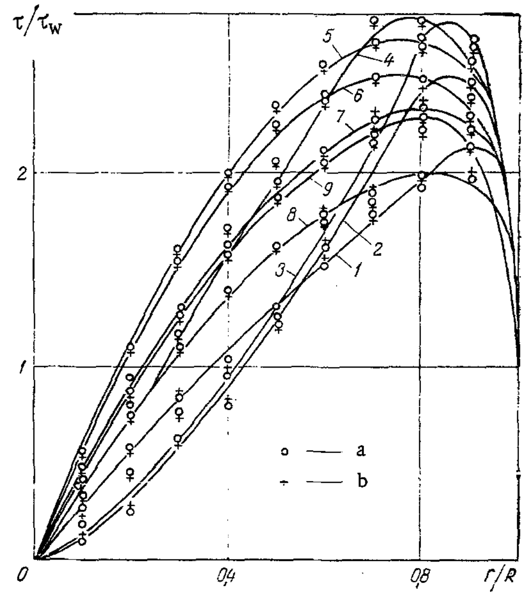

Fig. 5. Dependence of shear stress τ/τ_W on r/R and x/D: 1) $\frac{x}{D}=0$; 2) 3.0; 3) 4.5; 4) 6.0; 5) 7.5; 6) 9.0; 7) 10.5; 8) 12.0; 9) 13.5; a) $Re_0=67{,}200$; b) $Re_0=52{,}900$.

Curves plotted by the method of least squares are drawn through experimental points in order to determine the maximum axial velocity at the flow axis and also in order to find accurately interpolated values of the velocity for any value of r/R. The mean velocity for a given position is calculated by integrating the velocity profile.

The coefficient of momentum flow is calculated according to the equation

$$\beta = \frac{1}{\bar{u}^2}\int_0^1 u^2 d\,(r/R)^2. \tag{2}$$

The radial components of velocity are found by integrating the continuity equation in cylindrical coordinates from the pipe axis to a given radial position:

$$v = \frac{1}{4\rho\,(r/R)}\cdot\frac{\partial\left[\rho\int_0^{r/R} u d\,(r/R)^2\right]}{\partial\,(x/D)}. \tag{3}$$

The radial distribution of shear stresses in the flow is found from the equation

$$\tau = -\frac{(r/R)}{4}\cdot\frac{dP}{d\,(x/D)} - \frac{1}{2\,(r/R)}\left\{\frac{\partial\left[\rho\int_0^{r/R} u^2(r/R)\,d\,(r/R)\right]}{\partial\,(x/D)} + u\,\frac{\partial\left[\rho\int_0^{r/R} u\,(r/R)\,d\,(r/R)\right]}{\partial\,(x/D)}\right\}. \tag{4}$$

When (r/R) = 1, a value can be found from Eq. (4) for the magnitude of the shear stress at the wall $\tau = \tau_W$. The local coefficient of friction is determined from the correlation

$$\xi = \frac{8\tau_W}{\rho\bar{u}^2}. \tag{5}$$

The maximum v(r/R) calculation error occurring in finite sections of the porous channel does not exceed 8%. The error in determining τ_W may be as high as 10% and the error in finding ξ is not more than 12%.

Experiments show that the flow is a fully developed turbulent one when it enters the porous section of the pipe and corresponds to the conditions of turbulent flow in a smooth pipe. The variation in the relative pressure along the porous section is shown in Fig. 2. The nature of the variation in longitudinal velocity along the working section can be established from Fig. 3 which gives a family of velocity profiles for one of the sets of flow conditions (v_W = 0.63 m/sec). It follows from Fig. 3 that the longitudinal velocity by the wall is reduced sharply at the start of the porous pipe due to the outflow, while along the pipe axis it is altered only slightly. This type of outflow continues over a distance equal to about 8 pipe diameters. Thereafter, the outflow causes a reduction in velocity at the flow core and flow formation ceases approximately at the section equal to 10 diameters.

When $x/D \geq 10$ the dimensionless velocity profile becomes self-similar in terms of the x/D coordinate and is approximated within the limits of $0 \leq r/R \leq 0.95$ by the equation

$$\frac{u}{\bar{u}} = \frac{1.275^3}{1.275^2 + (r/R)^2}. \qquad (6)$$

The coefficient of momentum flow shown in Fig. 4 also reflects the significant variations in the axial velocity profiles which occur throughout an initial section of the pipe extending from the inlet to a distance equal in length to 10 diameters of the pipe. The magnitude of β rises at first until β = 1.03 when x/D = 7.5 and then falls tending at $x/D \geq 10$ to a self-similar value.

The results of calculating the shear stresses in the flow for various different transverse sections of the porous pipe are given in Fig. 5 in the form of the ratio of shear stress in a given radial position to shear stress at the wall in the same section. The flow spread and displacement of fluid mass in the transverse direction are accompanied by an increase in both the absolute value of τ and the ratio τ/τ_W.

It is clear that the distribution of shear stresses in the flow differs sharply from the normal linear correlation characteristic of a flow with continuous walls. The maximum shear stresses in the flow occur not at the wall, but shifted slightly toward the center of the flow. The maximum shift is observed when x/D = 9 and is $(y/R) \approx 0.3$. The local values of the relative shear stresses τ/τ_W are increased sharply until x/D = 6 and are then reduced gradually.

The experiments show that for a completely stabilized flow all the parameters characterizing the structure of a turbulent flow in the range under investigation are, in practice, self-similar in terms of the Reynolds number (Re). The dependence

$$\xi = 17.5 k_{\perp}, \qquad (7)$$

where $k_{\perp} = v_W/\bar{u}$ is the intensity of outflow, is proposed in [4] for calculating the coefficient of friction given an exponential law governing the variation in velocity in a porous channel.

In these experiments, however, the values of the coefficient of friction do not correspond to the calculated values found by using formula (7). Thus, the magnitude of the local coefficient of friction when the flow spreads through a porous wall is dependent not only on the intensity of outflow, but also on the law governing it.

In an investigation into the averaged hydraulic characteristics of a turbulent flow with a uniform outflow through a porous wall in pipes with a diameter D = 13.85 mm with a porosity ε_f = 0.1156 and ε_f = 0.5 in a range of variations in relative length $14.45 \leq (L/D) \leq 72.2$, the following calculation formula is obtained;

$$\xi = \xi_0 + 15.6 k_{\perp}^{1.27} + \frac{m}{k_{\perp}}\left(1 - \frac{k_{\perp_0}}{k_{\perp}}\right). \qquad (8)$$

Here m is the experimental coefficient dependent on the porosity and relative length of the pipe.

For porosity ε_f = 0.5

$$m = 0.0256 k_{\perp_0}^{0.435}, \qquad (9)$$

where

$$k_{\perp_0} = \frac{v_W}{\bar{u}_0} = \frac{1}{4\,(L/D)}.$$

The dependences proposed are in satisfactory agreement with the experimental data and can be used to determine the coefficient of friction in circular porous channels with a uniform spread of flow through a wall when $Re_0 > 10^4$.

NOTATION

p, static pressure; $p_x = p(x)$; ρ, density; $\bar{u}$, mean velocity in channel section; $u_x = u(x)$; v, radial velocity at wall; G, mass rate; L, total length of porous channel; x, coordinate; D, equivalent channel diameter; d, diameter of apertures in channel wall; ε_f, channel porosity; ξ, coefficient of friction resistance; Re, Reynolds number; β, coefficient of momentum flow; τ, shear stress.

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VELOCITY AND FRICTION STRESS DISTRIBUTIONS IN THE TURBULENT BOUNDARY LAYER ON A POROUS PLATE

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The turbulent viscosity above a permeable plate is determined by using the magnitude of the maximum turbulent friction in the boundary layer. Simple dependences are obtained for the velocity and friction stress distribution.

Theoretical investigations of the turbulent boundary layer on a porous wall, whose results are suitable for engineering utilization, are based, as a rule, on a two-layer flow scheme. The Newton formula is used for the friction stress in the laminar sublayer where viscous flow predominates, while the Prandtl (or Kármán) formula is applied in the turbulent core by using the characteristic mixing path length augmented by the distance from the wall. The friction stress distribution in the boundary layer is simultaneously taken as $\tau = \tau_w + \rho_w v_w u$ [1-4], according to which τ increases monotonically over the boundary-layer thickness during blowing. The mentioned hypotheses, which are used in theoretical methods, were subjected to criticism in experimental studies [5-7]. It is shown that the magnitude of the friction stress during blowing from the wall has a maximum within the boundary layer (for $\bar{u} \approx 0.65$) but the mixing path length increases with distance from the wall only near it.

A dependence for the turbulent viscosity over a permeable plate is proposed in this paper, which agrees with recent experimental results and thus permits later investigation of the boundary layer in the presence of physicochemical processes. Let us first derive an integral relation for the momentum in a form convenient for subsequent solution of the problem. Integrating the motion and continuity equations for the boundary layer on a plate between the limits 0 and y along the normal coordinate, we obtain

$$\frac{\partial}{\partial x}\left(\int_0^y \rho u^2 dy\right) = \tau - \tau_w - \rho v u, \qquad \frac{\partial}{\partial x}\left(\int_0^y \rho u dy\right) = \rho_w v_w - \rho v. \tag{1}$$

From the condition that the friction stress is zero on the outer boundary-layer limit, there follows from (1)

$$u_\infty^2 \frac{d}{dx}\int_0^\delta \rho \bar{u}(1-\bar{u})\, dy = \tau_w (1+B), \tag{2}$$

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